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Memorino on the '1/2 versus 3/2 puzzle' in $\bar{B} \to l\bar{\nu}X_c$ – a year later and a bit wiser

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Abstract. The OPE treatment that has been so successful in describing inclusive $\bar{B} \to l\bar{\nu}X_c$ decays yields sum rules (in particular the Uraltsev sum rule and its higher moments) implying the dominance of the P wave $j_q = 3/2$ charm states in X_c over their $j_q = 1/2$ counterparts. This prediction is supported by other general arguments as well as quark model calculations, which illustrate the OPE results, and by preliminary lattice findings. Its failure would indicate a significant limitation in our theoretical understanding of $\bar{B} \to l\bar{\nu}X_c$. Some experimental issues have been clarified since a preliminary version of this note had appeared; yet, the verdict on the composition of the final states beyond D, D^* and the two narrow $j_q = 3/2$ resonances remains unsettled. Establishing which hadronic configurations – $D/D^* + \pi, D/D^* + 2\pi, \ldots$ – contribute, what their quantum numbers are, and their mass distributions will require considerable experimental effort. We explain the theoretical issues involved and why a better understanding of them will be of considerable value. Having significant contributions from a mass continuum distribution below 2.5 GeV raises serious theoretical questions for which we have no good answer. Two lists are given, one with measurements that need to be done and one with items of theoretical homework. Some of the latter can be done by employing existing theoretical tools, whereas others need new ideas.

1 Outline

Both our theoretical and experimental knowledge on semileptonic B decays have advanced considerably over the last 15 years. This progress can be illustrated most strikingly by the recent success in extracting the value of $|V(cb)|$ with better than 2% accuracy from measurements of inclusive $\bar{B} \to l\bar{\nu}X_c$ transitions [2–4]. At the same time some potential problem of a rather subtle nature have emerged. One concerns $\mathrm{BR}(B \to l\nu X_c)$, which has been very well measured now [5]:

$$\mathrm{BR}\left(\bar{B}_d \to l^-\bar{\nu}X_c\right) = (10.33 \pm 0.28)\% \tag{1}$$

$$\mathrm{BR}\left(\bar{B}_u \to l^-\bar{\nu}X_c\right) = (10.99 \pm 0.28)\%\,. \tag{2}$$

While the ratio of these branching fractions is well understood in terms of the $\bar{B}_d$–B^- lifetime difference, their absolute scale falls below early predictions inferred from a heavy quark expansion (HQE). Yet those were based on values for the charm quark mass that appear too heavy now. Using smaller values of m_c (together with a more careful definition of heavy quark masses) and including some novel radiative corrections [7] enhances the rate for $b \to c\bar{c}s$ and thus lowers $\mathrm{BR}_{\mathrm{SL}}(B)$. It also enhances the charm content in the final state of B decays – in agreement with the data [8]. A conclusive theoretical analysis of $\mathrm{BR}_{\mathrm{SL}}(B)$ has not been performed yet, although the tools exist. Yet we do not suspect this observable to represent a real problem for the theory.

In this note we want to focus on the composition of the hadronic final state in semileptonic B decays beyond $\bar{B} \to l\bar{\nu}D/D^*$. Understanding the nature of the hadronic system in the final state – its quantum numbers as well as mass distributions – is important, since well-grounded theoretical expectations and predictions can and have been given on these issues. Heavy quark symmetry, which becomes an exact symmetry of QCD in the limit $m_Q \to \infty$, provides at least a convenient classification scheme. The S wave configurations D and D^* represent the ground states to be followed by four P wave $[c\bar{q}]$ excitations. In two of those the light degrees of freedom carry angular momentum $j_q = 3/2$, resulting in narrow resonances with spin 1 and 2. For the other two one has $j_q = 1/2$ leading to broad resonances with spin 0 and 1. Our theoretical understanding of the semileptonic B decays tells us rather unequivocally that the $j_q = 3/2$ states should be more abundant in the final states than their $j_q = 1/2$ counterparts. This

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prediction appears to be at variance with some data. We refer to this apparent conflict as the '1/2 versus 3/2 puzzle' [1]. One also expects smallish contributions from other hadronic configurations. There still seems to be some tension between data sets concerning the mass distribution of such additional contributions and their decay patterns. The aim of this note is to explain in a concise way the arguments involved in deriving the theoretical expectations and the consequences of their possible failure to stimulate further experimental as well as theoretical studies. The issues had caught our attention and led to the original 'memorino' (= short memo) more than a year ago [9]. While more data have been obtained since, and we have pondered the issues further, we find the problems now even more intriguing and in need of a resolution. The latter has to be driven by even more detailed analyses. A more appropriate name for this paper might now be 'memorone' (= long memo) – alas we decided to stick with the original moniker.

After giving an overview of the experimental situation for $B \to l\nu X_c$ in Sect. 2 we marshall the theoretical arsenal for treating those decays: the operator product expansion (OPE) in Sect. 3, the BT model in Sect. 4 and lattice QCD in Sect. 5 before adding other general arguments in Sect. 6; in Sect. 7 we undertake a more detailed comparison of the theoretical predictions and expectations with the existing data on $\bar{B} \to l\bar{\nu}D^{(*)} + \pi$s from ALEPH, BaBar, BELLE, CDF, DELPHI and D0; in Sect. 8 we comment on the corresponding expectations for nonleptonic B decays; finally in Sect. 9 we list needed homework for both theorists and experimentalists. We aim at being as concise as reasonably possible, while providing a guide through the literature for the more committed reader.

2 The data

About three quarters of the inclusive semileptonic B width are made up by the two channels $\bar{B} \to l\nu D/D^*$, for which a recent BaBar analysis finds [16]:

$$\frac{\Gamma(B^- \to l^-\bar{\nu}D)}{\Gamma(B^- \to l^-\bar{\nu}DX)} = 0.227 \pm 0.014 \pm 0.016\,,$$
$$\frac{\Gamma(B^- \to l^-\bar{\nu}D^*)}{\Gamma(B^- \to l^-\bar{\nu}DX)} = 0.582 \pm 0.018 \pm 0.030$$
$$\frac{\Gamma(\bar{B}_d \to l^-\bar{\nu}D)}{\Gamma(\bar{B}_d \to l^-\bar{\nu}DX)} = 0.215 \pm 0.016 \pm 0.013\,,$$
$$\frac{\Gamma(\bar{B}_d \to l^-\bar{\nu}D^*)}{\Gamma(\bar{B}_d \to l^-\bar{\nu}DX)} = 0.537 \pm 0.031 \pm 0.036\,.$$

(One expects $\Gamma(\bar{B} \to l^-\bar{\nu}DX)$ to saturate $\Gamma(\bar{B}_d \to l^-\bar{\nu}X_c)$ for all practical purposes, and this is completely consistent with observation.) This large dominance of the D and D^* final states, the ground states of heavy quark symmetry, represents actually the most direct evidence that charm quarks act basically like heavy quarks in B decays. This can be invoked to justify using the heavy quark classification already for charm and applying arguments based on the SV limit [17].

For the remainder we have

$$\frac{\Gamma(B^- \to l^-\bar{\nu}D^{**})}{\Gamma(B^- \to l^-\bar{\nu}DX)} = 0.191 \pm 0.013 \pm 0.019\,, \quad (3)$$
$$\frac{\Gamma(\bar{B}_d \to l^-\bar{\nu}D^{**})}{\Gamma(\bar{B}_d \to l^-\bar{\nu}DX)} = 0.248 \pm 0.032 \pm 0.030\,, \quad (4)$$

where D^{**} denotes any $D^{(*)}n\pi$ combination that is not a D^*. The apparent sizable difference in the central values of these two ratios is in conflict with theoretical expectations based on isospin symmetry. For the latter imposes practically equal rates for the corresponding semileptonic channels of B^- and $\bar{B}_d$. We view this difference as due to a statistical fluctuation or a systematic bias. Averaging over the B^- and $\bar{B}_d$ rates yields [16]

$$\frac{\Gamma(\bar{B} \to l^-\bar{\nu}D)}{\Gamma(\bar{B} \to l^-\bar{\nu}DX)} = 0.221 \pm 0.012 \pm 0.006\,,$$
$$\frac{\Gamma(\bar{B} \to l^-\bar{\nu}D^*)}{\Gamma(\bar{B} \to l^-\bar{\nu}DX)} = 0.572 \pm 0.017 \pm 0.016\,,$$
$$\frac{\Gamma(\bar{B} \to l^-\bar{\nu}D^{**})}{\Gamma(\bar{B} \to l^-\bar{\nu}DX)} = 0.197 \pm 0.013 \pm 0.013\,. \quad (5)$$

BELLE finds completely consistent branching ratios for $B^- \to l^-\bar{\nu}D^0/D^{*0}$ and $\bar{B}_d \to l^-\bar{\nu}D^+$. Yet their number for $\bar{B}_d \to l^-\bar{\nu}D^{*+}$ appears on the low side compared with the expectation based on isospin. As before we take that as a sign that the data are not completely mature yet, rather than as a real effect.

The numbers in (5) are consistent with the HFAG averages [5]:

$$\mathrm{BR}\left(\bar{B}_d \to l^-\bar{\nu}D^+\right) = (2.08 \pm 0.18)\%\,,$$
$$\mathrm{BR}\left(\bar{B}_d \to l^-\bar{\nu}D^{*+}\right) = (5.29 \pm 0.19)\%\,,$$
$$\mathrm{BR}\left(\bar{B}_d \to l^-\bar{\nu}D^{**+}\right) = (2.8 \pm 0.3)\%\,, \quad (6)$$

where the D^{**} rate is inferred by subtracting the D and D^* rates from the total semileptonic rate. Another even more recent BABAR analysis [6] finds numbers manifestly consistent with isospin invariance:

$$\mathrm{BR}\left(B^- \to l^-\bar{\nu}D^0\right) = (2.33 \pm 0.09_{\mathrm{stat}} \pm 0.09_{\mathrm{syst}})\,\%\,, \quad (7)$$
$$\mathrm{BR}\left(B^- \to l^-\bar{\nu}D^{*0}\right) = (5.83 \pm 0.15_{\mathrm{stat}} \pm 0.30_{\mathrm{syst}})\,\%\,, \quad (8)$$
$$\mathrm{BR}\left(\bar{B}_d \to l^-\bar{\nu}D^+\right) = (2.21 \pm 0.11_{\mathrm{stat}} \pm 0.12_{\mathrm{syst}})\,\%\,, \quad (9)$$
$$\mathrm{BR}\left(\bar{B}_d \to l^-\bar{\nu}D^{*+}\right) = (5.49 \pm 0.16_{\mathrm{stat}} \pm 0.25_{\mathrm{syst}})\,\%\,. \quad (10)$$

The ratio of the corresponding branching ratios is fully consistent with the B^-–$\bar{B}_d$ lifetime ratio.

The four P wave excitations $D^{3/2}_{1,2}$ and $D^{1/2}_{0,1}$ are obvious candidates to provide D^{**} contributions. ALEPH [34] has reconstructed D^{**} states decaying into $D^{(*)}\pi^{\pm}$. The authors of this reference did not observe a significant excess of events over the expected background in $D^{(*)+}\pi^+$ or

$D^0\pi^-$ combinations (called 'wrong sign'). From the measured rate of 'right sign' combinations and assuming that only D^{**} decaying to $D^{(*)}\pi$ contribute (to correct for channels with a missing π^0) they get (with $\text{Prob}(b \to B) = (39.7 \pm 1.0)\%$)

$$\text{BR}\left(\bar{B} \to l\bar{\nu}D^{**} \to l\bar{\nu}D^{(*)}\pi\right) = (2.2 \pm 0.3 \pm 0.3)\,\% \,. \quad (11)$$

Assuming the $D_1^{3/2}$ state to decay only into $D^*\pi$ they find from their D^{**} sample

$$\text{BR}\left(\bar{B} \to l\bar{\nu}D_1^{3/2}\right) = (0.70 \pm 0.15)\% \,, \quad (12)$$

$$\text{BR}\left(\bar{B} \to l\bar{\nu}D_2^{3/2}\right) < 0.2\% \,. \quad (13)$$

DELPHI has published a re-analysis of their data [35], superseding their previous study [36]. Assuming only $D^{(*)}\pi$ to contribute (to correct for channels with a missing π^0) one obtains

$$\text{BR}\left(\bar{B} \to l\bar{\nu}D^{**} \to l\bar{\nu}D^{(*)}\pi\right) = (2.7 \pm 0.7 \pm 0.2)\% \,, \quad (14)$$

with clear evidence for two narrow states tentatively identified with $D^{3/2}$:

$$\text{BR}\left(\bar{B} \to l\bar{\nu}D_1^{3/2}\right) = (0.56 \pm 0.10)\% \,,$$

$$\text{BR}\left(\bar{B} \to l\bar{\nu}D_2^{3/2}\right) = (0.30 \pm 0.08) \,. \quad (15)$$

The D0 collaboration has measured production rates of narrow D^{**} states in the decay $\bar{B} \to \mu^-\bar{\nu}D^*\pi$. Assuming $\text{BR}(D_1^{3/2} \to D^*\pi) = 100\%$ and $\text{BR}(D_2^{3/2} \to D^*\pi) = (30 \pm 6)\%$ they obtain [37]

$$\text{BR}\left(\bar{B} \to \mu^-\bar{\nu}D_1^{3/2}\right) = (0.33 \pm 0.06)\% \,,$$

$$\text{BR}\left(\bar{B} \to \mu^-\bar{\nu}D_2^{3/2}\right) = (0.44 \pm 0.16)\% \,. \quad (16)$$

These three sets of data agree in pointing to

$$\text{BR}\left(\bar{B} \to l^-\bar{\nu}D_{1,2}^{3/2}\right) \sim 0.8\%\text{–}1\% \,, \quad (17)$$

yet they do not paint a clear picture on the relative strength of $\bar{B} \to l^-\bar{\nu}D_2^{3/2}$ versus $\bar{B} \to l^-\bar{\nu}D_1^{3/2}$.

The observed rates for $\bar{B} \to l^-\bar{\nu}D/D^*/D^{3/2}$ are in pleasing agreement with the theoretical predictions described below. The fact that they do not quite saturate $\Gamma_{\text{SL}}(\bar{B})$ is not surprising. Even so one would like to learn from the data what these additional final states are, in particular what their distribution in mass is. If they populate the range above 2.5 GeV, then we have at least candidates for them. If, however, they fall mostly below 2.5 GeV, then we can come up at best with rather exotic explanations. These are subtle questions concerning smallish rates. Yet answering them will teach us important and potentially very surprising lessons on non-perturbative dynamics. This will be explained in Sect. 7 after having marshaled the theoretical predictions and expectations.

3 The OPE treatment

While the OPE allows one to describe inclusive transitions, no systematic extension to exclusive modes has been given so far. Yet even so, the OPE allows us to place important constraints on some exclusive rates: $\bar{B} \to l\bar{\nu}D^*$, $l\bar{\nu}D$ (the latter involving the 'BPS' approximation on which we comment later) are the most topical and elaborated examples [10–12].

OPE results can be given also for subclasses of inclusive transitions due to various sum rules [13–15] that can genuinely be derived from QCD; hence one can infer constraints on certain exclusive contributions. Those can be formulated most concisely when one adopts the heavy quark symmetry classification scheme also for the charm system in the final state of semileptonic B decays.

In the limit $m_Q \to \infty$ one has heavy quark symmetry controlling the spectroscopy for mesons as follows. The heavy quark spin decouples from the dynamics, and the hadrons can be labeled by their total spin S together with the angular momentum j_q carried by the light degrees of freedom, namely the light quarks and the gluons. The pseudoscalar and vector mesons D and D^* then form the ground states of heavy quark symmetry in the charm sector with $j_q = 1/2$. The first excited states are four P wave configurations, namely two with $j_q = 3/2$ and $S = 2, 1$ – $D_2^{3/2}$, $D_1^{3/2}$ – and two with $j_q = 1/2$ and $S = 1, 0$ – $D_1^{1/2}$, $D_0^{1/2}$; the two 3/2 states are narrow resonances and the two 1/2 states wide ones. Then there are higher states still, namely radial excitations and higher orbital states; furthermore there are charm final states that cannot be properly called a hadronic resonance but are D/D^* combinations with any number of pions etc. carrying any allowed J^{PC} quantum numbers.

The P wave states $D_{2,1}^{3/2}$ and $D_{1,0}^{1/2}$ are obvious candidates for D^{**}, but they should not saturate it completely. One actually expects QCD radiative corrections to populate the higher hadronic mass region above the prominent resonances through a smooth spectrum dual to a superposition of broad resonances.

Up to small isospin breaking effects one predicts these semileptonic rates to be the same. The usual Isgur–Wise function $\xi(w)$ is the core element in describing $\bar{B} \to l\bar{\nu}D/D^*$. It can be generalized to describe also the production of excited charm final states in semileptonic B decays: $\tau_{1/2[3/2]}^{(n)}(w_n)$ with $w_n = v_B \cdot v_{D^{(n)}}$ is the amplitude for $\bar{B} \to l\bar{\nu}D_{1/2[3/2]}^{(n)}$, where $D_{1/2[3/2]}^{(n)}$ denotes a hadronic system with open charm carrying $j_q = 1/2[3/2]$ and label n; it does not need to be a bona fide resonance.

Various sum rules can be derived from QCD proper relating the moduli of these amplitudes and powers of the excitation energies $\epsilon^{(n)} \equiv M_{D^{(n)}} - M_D$ to the heavy quark parameters. Adopting the so-called 'kinetic scheme', as we will throughout this note, one obtains in particular [13]

$$\frac{1}{4} = -\sum_n \left|\tau_{1/2}^{(n)}(1)\right|^2 + \sum_m \left|\tau_{3/2}^{(m)}(1)\right|^2 , \quad (18)$$

$$\mu_\pi^2(\mu)/3 = \sum_n^{\epsilon^{(n)} \le \mu} \left(\epsilon_{1/2}^{(n)}\right)^2 \left|\tau_{1/2}^{(n)}(1)\right|^2 + 2 \sum_m^{\epsilon^{(m)} \le \mu} \left(\epsilon_{3/2}^{(m)}\right)^2 \left|\tau_{3/2}^{(m)}(1)\right|^2 , \quad (19)$$

$$\mu_G^2(\mu)/3 = -2 \sum_n^{\epsilon^{(n)} \le \mu} \left(\epsilon_{1/2}^{(n)}\right)^2 \left|\tau_{1/2}^{(n)}(1)\right|^2 + 2 \sum_m^{\epsilon^{(m)} \le \mu} \left(\epsilon_{3/2}^{(m)}\right)^2 \left|\tau_{3/2}^{(m)}(1)\right|^2 , \quad (20)$$

where the summations go over all hadronic systems with excitation energies $\epsilon^{(n,m)} \le \mu$.[1] The sum rules relate the observables $\mu_\pi^2(\mu)$ and $\mu_G^2(\mu)$ that describe the fully inclusive $\bar{B} \to l^-\bar{\nu}X_c$ transitions with sums over subclasses of exclusive modes. For the purpose of these sum rules it is irrelevant whether these hadronic systems are bona fide resonances or not; what matters is that they are P wave configurations with $j_q = 3/2$ or $1/2$.

These sum rules allow us to make both general qualitative as well as (semi-) quantitative statements. On the qualitative level we learn unequivocally that the '3/2' transitions have to dominate over the '1/2' ones, as can be read off from (18) and (20). Furthermore we know that

$$\mu_\pi^2(\mu) \ge \mu_G^2(\mu) \quad (21)$$

has to hold for any μ [17], as is obvious from (19) and (20). On the *quantitative* level it is not a priori clear, at which scale μ these sum rules are saturated and by which kind of states. We will address these issues below.

We have learnt a lot about the numerical values of the heavy quark parameters: the most accurate value for the chromomagnetic moment μ_G^2 can be deduced from the B^*–B hyperfine mass splitting:

$$\mu_G^2(1\,\mathrm{GeV}) = (0.35 \pm 0.03)\,\mathrm{GeV}^2 . \quad (22)$$

The analyses of [2–4] based on a comprehensive study of energy and hadronic mass moments in $\bar{B} \to l\bar{\nu}X_c$ yield the following values:

$$\mu_G^2(1\,\mathrm{GeV}) = \begin{cases} 0.297 \pm 0.024|_{\mathrm{exp}} \pm 0.046|_{\mathrm{HQE}}\,\mathrm{GeV}^2 & [2]\,, \\ 0.358 \pm 0.060|_{\mathrm{fit}} \pm 0.003|_{\delta\alpha_S}\,\mathrm{GeV}^2 & [3]\,, \\ 0.330 \pm 0.042|_{\mathrm{exp}} \pm 0.043|_{\mathrm{theo}}\,\mathrm{GeV}^2 & [4]\,, \end{cases} \quad (23)$$

$$\mu_\pi^2(1\,\mathrm{GeV}) = \begin{cases} 0.401 \pm 0.019|_{\mathrm{exp}} \pm 0.035|_{\mathrm{HQE}}\,\mathrm{GeV}^2 & [2]\,, \\ 0.557 \pm 0.091|_{\mathrm{fit}} \pm 0.013|_{\delta\alpha_S}\,\mathrm{GeV}^2 & [3]\,, \\ 0.471 \pm 0.034|_{\mathrm{exp}} \pm 0.062|_{\mathrm{theo}}\,\mathrm{GeV}^2 & [4]\,. \end{cases} \quad (24)$$

These experimental numbers are consistent with each other and with the predictions of (22) and (21) without the latter having been imposed. All three determinations of $\mu_\pi^2(1\,\mathrm{GeV})$ are within one sigma of $0.45\,\mathrm{GeV}^2$, which we will use as a reference point for our subsequent considerations:

$$\mu_\pi^2(1\,\mathrm{GeV})|_{\mathrm{ref}} = 0.45\,\mathrm{GeV}^2 . \quad (25)$$

For the lowest excitation energies we have

$$\epsilon_{3/2}^{(0)} \sim 450\,\mathrm{MeV}\,, \quad \epsilon_{1/2}^{(0)} \sim (300-500)\,\mathrm{MeV}\,; \quad (26)$$

these values for $\epsilon_{1/2}^{(0)}$ allow for '1/2' states to be both lighter and heavier than the narrow '3/2' states.

From (19) and (20)) one obtains

$$\mu_G^2(\mu) = 6 \sum_n^{\epsilon^{(n)} \le \mu} \left(\epsilon_{3/2}^{(n)}\right)^2 \left|\tau_{3/2}^{(n)}(1)\right|^2 - \frac{2}{3}\left(\mu_\pi^2(\mu) - \mu_G^2(\mu)\right) , \quad (27)$$

$$\mu_\pi^2(\mu) - \mu_G^2(\mu) = 9 \sum_n^{\epsilon^{(n)} \le \mu} \left(\epsilon_{1/2}^{(n)}\right)^2 \left|\tau_{1/2}^{(n)}(1)\right|^2 \quad (28)$$

as convenient expressions to read off natural 'scenarios' for the implementation of the OPE description and its sum rules.

One can reasonably assume these sum rules to be saturated approximately by the lowest states $n = 0$ for $\mu \le 1\,\mathrm{GeV}$. This rule of thumb (not to be confused with sum rules) is based on general experience with sum rules and on considerations of how $\mu_\pi^2(\mu)$ and $\mu_G^2(\mu)$ vary with the scale μ. It does not mean that the various sum rules would saturate to the same degree at a given μ, for they reflect different dynamical situations.

Using (25) we then infer from (27)

$$\tau_{3/2}^{(0)}(1) \sim 0.6 \quad (29)$$

and from (28)

$$\tau_{1/2}^{(0)}(1) \le 0.14\text{–}0.32\,, \quad (30)$$

which are reasonable numbers as our subsequent considerations will illustrate. For $\mu_\pi^2(1\,\mathrm{GeV}) = 0.4\,\mathrm{GeV}^2$ one has $\tau_{1/2}^{(0)}(1) \le 0.1$–$0.2$ and $\tau_{3/2}^{(0)}(1) \sim 0.56$. On the other hand $\mu_\pi^2(1\,\mathrm{GeV}) = 0.55$ allows for a sizeable production rate of the lowest '1/2' state – $\tau_{1/2}^{(0)}(1) \le 0.2$–$0.45$ – although it does not enforce it, and $\tau_{3/2}^{(0)}(1) \sim 0.63$. These numbers for $\tau_{3/2}^{(0)}(1)$ should also be seen more like an upper bound, since the assumed saturation of the sum rules can hardly be exact.

Imposing also Uraltsev's sum rule, (18), with (approximate) saturation assumed for the lowest states

[1] The sum rule of (18) does not require a cut-off or normalization scale μ, as is already implied by its left hand side [13].

$\left(\left|\tau_{1/2}^{(0)}(1)\right|^2 \simeq \left|\tau_{3/2}^{(0)}(1)\right|^2 - \frac{1}{4}\right)$ leads to

$$\tau_{3/2}^{(0)}(1)|_{\rm SR} \sim 0.6\,, \quad \tau_{1/2}^{(0)}(1)|_{\rm SR} \sim 0.32\,; \tag{31}$$

i.e., very much the upper end of (30) inferred from $\mu_\pi^2(1\,\text{GeV}) = 0.45\,\text{GeV}^2$ and pointing to $\epsilon_{1/2}^{(0)} \sim 300\,\text{MeV}$, i.e. a relatively low mass for the '1/2' states. A scenario with a low $\mu_\pi^2(1\,\text{GeV}) = 0.4\,\text{GeV}^2$ is, however, hardly compatible with it, whereas a high $\mu_\pi^2(1\,\text{GeV}) = 0.55\,\text{GeV}^2$ can be accommodated with $\epsilon_{1/2}^{(0)} \sim 450\,\text{MeV}$.

While these numbers are reasonable, one cannot rule out significant contributions from higher states like the first radial excitations of the P wave states. We will address such a scenario in Sect. 4.

In summary we have the following.

- The OPE treatment leads to the general prediction that among P wave configurations production of '3/2' states dominates over that of '1/2' states in semileptonic B decays and that the former yields a significant contribution.
- Approximate saturation of the sum rules by the P wave states represents a scenario consistent with the data and leads to semi-quantitative estimates for the degree of the dominance of '3/2' over '1/2' production.
- At the same time there is no reason from the OPE to expect that D, D^* and the two narrow $D_{3/2}$ states saturate the semileptonic width. We know that the OPE treatment describes the hadronic mass moments in semileptonic B decays very successfully [2–4], and the data clearly show $\text{BR}(\bar{B} \to l^-\bar{\nu}D_X) \sim 1$–$2\%$ with $D_X \neq (D, D^*, D_{1,2}^{(3/2)})$. Already on general grounds one would not expect D_X to be mostly a narrow resonance. More specifically one can start from the numbers in (6) and split the $\bar{B} \to l^-\bar{\nu}D^{**}$ contribution into two components with $\text{BR}(\bar{B} \to l^-\bar{\nu}D^{(3/2)}) = 0.8\%$ and $\text{BR}(\bar{B} \to l^-\bar{\nu}D_X) = 2\%$. From the hadronic mass moments determined in the kinetic scheme [2, 3] one can then infer by matching what the hadronic mass moments for the D_X contribution have to be. One typically finds $\langle M(D_X)\rangle \sim 2.4$–$2.6\,\text{GeV}$ with a spread of about 200 MeV. DELPHI has inferred the mass moments of the D^{**} contributions and found a central value of 2.5 GeV with a spread of 230 MeV.
  Then the question arises of what makes up this D_X contribution. Some broad '3/2' configuration presumably of a non-resonant nature? Or states that do not contribute to the sum rules like $J^P = 0^-, 1^-$ states? Radial excitations would fit this bill, yet run counter to arguments to be discussed in Sect. 7.
- No reliable prediction on their decay patterns – i.e. whether they yield $D^{(*)}\pi$ or $D^{(*)}2\pi$ etc. – can be inferred from the OPE treatment per se.

4 The BT model

Based on the OPE treatment alone one cannot be more specific numerically. To go further one relies on quark models for guidance. The dominance of the '3/2' over the '1/2' states emerges naturally in all quark models obeying known constraints from QCD as well as Lorentz covariance. This can be demonstrated explicitly with the Bakamjian–Thomas covariant quark model [19, 20], which satisfies heavy quark symmetry and the Bjorken as well as spin sum rules referred to above. It allows us to determine the masses of various charm excitations and to compute the production rates in semileptonic [21–23] as well as nonleptonic B decays [22]. The BT model provides a quantitative illustration of the heavy quark limit, in particular concerning the sum rule of (18). One finds

$$\tau_{1/2}^{(0)}(1)\Big|_{\rm BT} = 0.22\,, \tag{32}$$

$$\tau_{3/2}^{(0)}(1)\Big|_{\rm BT} = 0.54 \tag{33}$$

together with predictions for the slopes. These values are fully consistent with the estimates given above. For the semileptonic modes the BT model yields

$$\text{BR}\left(\bar{B} \to l\bar{\nu}D\right) = (1.95 \pm 0.10)\,\%\,, \tag{34}$$

$$\text{BR}\left(\bar{B} \to l\bar{\nu}D^*\right) = (5.90 \pm 0.20)\,\%\,, \tag{35}$$

$$\text{BR}\left(\bar{B} \to l\bar{\nu}D_2^{3/2}\right) = \left(0.63^{+0.3}_{-0.2}\right)\%\,, \tag{36}$$

$$\text{BR}\left(\bar{B} \to l\bar{\nu}D_1^{3/2}\right) = \left(0.40^{+0.12}_{-0.14}\right)\%\,, \tag{37}$$

$$\text{BR}\left(\bar{B} \to l\bar{\nu}D_1^{1/2}\right) = (0.06 \pm 0.02)\,\%\,, \tag{38}$$

$$\text{BR}\left(\bar{B} \to l\bar{\nu}D_0^{1/2}\right) = (0.06 \pm 0.02)\,\%\,. \tag{39}$$

The following basis and features of these predictions should be noted.

- The predictions for $\bar{B} \to l\bar{\nu}D/D^*$ are based on the following parametrization of the Isgur–Wise function: $\xi(w) = \left(\frac{2}{w+1}\right)^{2\rho^2}$, where ρ^2 denotes its slope. For the latter we have used the value from [26]. The predictions agree with the data.
- The branching ratios for the P wave states and the theoretical uncertainties are obtained by using the BT model value for the form factors for $w = 1$ and allowing for a $\pm 50\%$ variation in the slope given by the BT model. This is the origin of the large relative errors in the predicted rates. We find a strong dominance of '3/2' over '1/2' production for the P wave states as inferred already from the sum rules.
- These branching ratios add up to $9.00 \pm 0.40\%$ and thus fall short of saturating the observed $\Gamma_{\rm SL}(B)$, (2). Such a deficit is not surprising as argued before. The more specific question is whether the BT model can account for the required additional width. The answer is not known yet. What can be said is that the hadronic mass for these extra contributions cannot be much lower than 2.6 GeV and that a sizable number of channels might be involved.
- The model as it is does not allow one to compute $1/m_Q$ corrections; i.e., effectively it treats the $m_Q \to \infty$ limit, since only then it is covariant.

With an explicit quark model one can address also higher states. The BT model finds for the first radial excitations of the '3/2' and '1/2' P wave states still sizable amplitudes $\tau_{3/2}^{(1)} \simeq 0.21$ and $\tau_{1/2}^{(1)} \simeq 0.20$. While these states enhance $\mu_\pi^2(1\ \mathrm{GeV})$ significantly, their contributions to $\Gamma_{\mathrm{SL}}(B)$ are still found to be insignificant.

5 Lattice QCD

In principle the two form factors $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$ can be computed in a straightforward way using the HQET equation of motion $(v \cdot D)\, h_v = 0$ [27]:

$$
\begin{aligned}
&{}_v\left\langle 0^+ \left|\bar{h}_v \gamma^i \gamma^5 D^j h_v\right| 0^-\right\rangle_v = \mathrm{i}g^{ij}\tau_{1/2}(1)\left(\overline{\Lambda}_{0^+} - \overline{\Lambda}_{0^-}\right), \\
&{}_v\left\langle 2^+ \left|\bar{h}_v \left(\frac{\gamma^i\gamma^5 D^j + \gamma^j\gamma^5 D^i}{2}\right) h_v\right| 0^-\right\rangle_v \\
&\qquad = -\mathrm{i}\sqrt{3}\epsilon^{*ij}\tau_{3/2}(1)\left(\overline{\Lambda}_{2^+} - \overline{\Lambda}_{0^-}\right), \qquad (40)
\end{aligned}
$$

where $v = (1, \mathbf{0})$ is the heavy quark velocity, ϵ^* the polarization tensor of the 2^+ state and $\overline{\Lambda}_{J^P}$ the dominant term in the OPE expression for the J^P heavy-light meson binding energy. On the lattice the covariant derivative D_i applied to the static quark field $h(\mathbf{x}, t)$ is expressed as $D_i h(\mathbf{x}, t) \to \frac{1}{2a}\Big(U_i(\mathbf{x}, t)h(\mathbf{x}+\hat{i}, t) - U_i^\dagger(\mathbf{x}-\hat{i}, t)h(\mathbf{x}-\hat{i}, t)\Big)$; $U_i(\mathbf{x}, t)$ denotes the gauge link. One calculates as usual the two-point functions $C^2_{J^P}(t) = \langle 0|O_{J^P}(t)O^\dagger_{J^P}(0)|0\rangle$, the three-point functions $C^3_{J^P,0^-}(t_1, t_2) = \langle 0|O_{J^P}(t_2)O_\Gamma(t_1) O^\dagger_{0^-}(0)|0\rangle$ and $\langle J^P|O_\Gamma|0^-\rangle \propto R(t_1, t_2) = \frac{C^3_{J^P,0^-}(t_1,t_2)}{C^2_{0^-}(t_1)C^2_{J^P}(t_2-t_1)}$.

Alas, numerical complications appear, because orbital as well as radial excitations can contribute. To extract properly the matrix element for the P wave state $\langle J^P|O_\Gamma|0^-\rangle$, one has to effectively suppress the coupling of radial excitations (with quantum numbers $n > 1, J^P$) to the vacuum. This can be achieved by choosing an appropriate interpolating field O_{J^P} such that $\langle n > 1 J^P|O_{J^P}|0\rangle = 0$ holds or by having huge statistics to diminish statistical fluctuations at large times (where the fundamental state is no more contaminated by radial excitations). This poses a problem in particular for the 2^+ state, for which the usual interpolating field seems to couple also the first radial excitation quite strongly to the vacuum. Moreover, reaching the required stability of $R(t_1, t_2)$ as a function of t_2 poses a serious challenge. Hopefully all to all propagators technology will be of great help, as it has already proved to be in studies of the static-light spectrum [28, 29] and in the determination of hadronic matrix elements [30].

We will need very careful and dedicated lattice studies to obtain meaningful and reliable results for $\tau_{3/2,1/2}$. As an already highly relevant intermediate step one can concentrate first on $\tau_{1/2}$ to see whether lattice QCD confirms its suppression as inferred from both the sum rules and the BT model. A preliminary study in the quenched approximation with $\beta = 6.0(a^{-1} = 2\ \mathrm{GeV}^{-1})$ and $m_q \simeq m_s$ yields [31–33]

$$\tau_{1/2}^{(0)}(1)\Big|_{\mathrm{LQCD}} \sim 0.41 \pm 0.05\,, \qquad (41)$$

$$\tau_{3/2}^{(0)}(1)\Big|_{\mathrm{LQCD}} \sim 0.57 \pm 0.10\,, \qquad (42)$$

where only the statistical errors are given. Again we find dominance of the '3/2' over the '1/2' amplitude even in numerical agreement with the values inferred from the sum rules, see (31); $\tau_{1/2}^{(0)}(1)|_{\mathrm{LQCD}}$ appears to be significantly larger than the BT estimate in (33).

Apart from unquenching and lowering the value of m_q one can improve and refine this analysis also by simulating a non-static charm quark, i.e. applying HQET to the B meson only. This would allow one to evaluate $1/m_c$ corrections. However, the first improvement has to be done with considerable care: while unquenched simulations with light quark masses lower than $m_s/5$ (i.e. a corresponding pseudoscalar Goldstone boson mass lower than 300 MeV) have become customary now, one has to worry about a possible mixing between a D^{**} resonance and a $D\pi$ state.

6 Two other general arguments on $|\tau_{1/2}/\tau_{3/2}|^2$

The numerics of the theoretical predictions on semileptonic B decays given above have to be taken cum grano salis. Yet their principal feature – the preponderance of '3/2' over '1/2' states – has to be taken very seriously, since they are a general consequence of the OPE treatment. It is further supported by two rather general observations that point in the same direction as the detailed theoretical considerations given above.

- When interpreting data one should keep in mind that the contributions of $D^{1/2}_{1,0}$ to $\Gamma(\bar{B} \to l\bar{\nu}D^{**})$ are suppressed relative to those from $D^{3/2}_{2,1}$ by a factor of two to three due to kinematics [21]. Thus one finds for reasonable values of $\tau_{1/2}^{(0)}$ that $\Gamma(\bar{B} \to l\bar{\nu}D^{1/2})$ falls below $\Gamma(\bar{B} \to l\bar{\nu}D^{3/2})$ by one order of magnitude, as illustrated above; see (38) and (39). For the two widths to become comparable, one would need a greatly enhanced $\tau_{1/2}^{(0)}$.
- There is a whole body of evidence showing that in so-called class I nonleptonic B decays like $\bar{B}_d \to D^{(*)+}\pi^-$ naive factorization provides a very decent description of the data. Invoking this ansatz also for $\bar{B}_d \to D^{**+}\pi^- \to D^{(*)0}\pi^+\pi^-$ one infers from BELLE's data [24] that the production of '1/2' states appears to be strongly suppressed relative to that for '3/2' ones. It implies that $|\tau_{1/2}/\tau_{3/2}|^2$ is small and certainly less than unity. The same feature is found in more recent measurements from BABAR [25].[2] This agrees with the theoretical expectations described before; more importantly it shows

[2] The comparison of the theoretical predictions with the measured rates is not straightforward, since the data are given

in a rather model independent way that there is no large unexpected enhancement of $|\tau_{1/2}|$. Those values also allow one to saturate the sum rule of (18) within errors already with the $n = 0$ states.
The form factors are actually probed at $w = 1.3$ in this nonleptonic transition; yet a natural functional dependence on w supports this conclusion to hold for $1 \leq w \leq 1.3$ in semileptonic channels.

These arguments are based on the heavy quark mass limit. The as yet unknown finite mass corrections could modify these conclusions somewhat.

7 Detailed comparison with the data on semileptonic B decays

Different experiments and theoretical treatments agree on

- $\Gamma(\bar{B} \to l^-\bar{\nu}X_c)$ being dominated by the two modes $\bar{B} \to l^-\bar{\nu}D/D^*$;
- with $D^{(3/2)}$ final states providing about 10% to it, and
- $\mathrm{BR}(\bar{B} \to l^-\bar{\nu}X_c) \simeq (1–2)\%$ has to come from other hadronic configurations D_X.

The question still open concerns the nature of this last component D_X.

At first sight one might wonder why one should worry about the identification of channels that sum up to no more than 2% in overall branching ratio. Yet they constitute 10–20% of all semileptonic transitions, and – maybe more importantly – theory makes quite non-trivial statements about them. We can learn important lessons about non-perturbative dynamics, even if those predictions are refuted by experiment.

Theory makes the rather robust prediction that it cannot come from the broad P wave states $D^{(1/2)}$. The OPE framework by itself can accommodate all three features listed above, as long as $D^{(1/2)}$ is insignificant in the third item. It points to hadronic contributions that are broad in mass without a firm prediction on their average mass – both $\langle M(D_X)\rangle \leq 2.4$ GeV or > 2.5 GeV seem a priori feasible – or their decay patterns; i.e. $D^{(*)}\pi$ versus $D^{(*)}\pi\pi$ (versus $D^{(*)}\eta$ etc.).

Both ALEPH and DELPHI can account for all of $\Gamma(\bar{B} \to l^-\bar{\nu}X_c)$ with $\bar{B} \to l^-\bar{\nu}D/D^*$ and $\bar{B} \to l^-\bar{\nu}D^{**} \to l^-\bar{\nu}D^{(*)}\pi$; see (11) and (14) with no established signal for $D^{(*)}\pi\pi$ states contributing. ALEPH places relatively tight bounds on higher combinations from the observed number of 'wrong sign' combinations:

$$\mathrm{BR}\left(\bar{B} \to l\bar{\nu}D^*\pi\pi\right) \leq 0.35\%\,,$$
$$\mathrm{BR}\left(\bar{B} \to l\bar{\nu}D\pi\pi\right) \leq 0.9\% \quad (90\%\,\mathrm{C.L.})\;; \qquad (43)$$

DELPHI's bounds are less tight:

$$\mathrm{BR}\left(\bar{B} \to l\bar{\nu}D^*\pi\pi\right) \leq 1.2\%\,, \quad \mathrm{BR}\left(\bar{B} \to l\bar{\nu}D\pi\pi\right) \leq 1.3\%\,. \qquad (44)$$

as products of production and decays branching ratios, and one has to use the model to calculate both. The BABAR analysis also lumps together the decays of more than one state.

Considering that 1^+ D^{**} can decay into $D\pi\pi$ and analyzing the $D\pi$ mass distribution, DELPHI fits a value of $(19 \pm 13)\%$ for this component. In this analysis of the hadronic mass moments such a possibility has been included with $\mathrm{BR}(\bar{B} \to l\bar{\nu}D\pi\pi) = (0.36 \pm 0.27)\%$. This turns out to be the dominant systematic uncertainty in this hadronic mass moment measurement.

DELPHI found a significant rate for producing a broad hadronic mass distribution in $D^{(*)}\pi$:

$$\mathrm{BR}\left(\bar{B} \to l\bar{\nu}D_{\text{“}1\text{”}}\right) = (1.24 \pm 0.25 \pm 0.27)\%\,,$$
$$\mathrm{BR}\left(\bar{B} \to l\bar{\nu}D_{\text{“}0\text{”}}\right) = (0.65 \pm 0.69)\%\,. \qquad (45)$$

If the broad contributions were indeed to be identified with the $D_{1,0}^{1/2}$ as already implied in (45) – an a priori reasonable working hypothesis – one would have a clear cut and significant conflict with the OPE expectations as well as the numerically more specific BT model predictions; see (38) and (39). For DELPHI's data would yield $\Gamma(\bar{B} \to l\bar{\nu}D^{1/2}) > \Gamma(\bar{B} \to l\bar{\nu}D^{3/2})$. This conflict has been referred to as the '1/2 > 3/2 puzzle' [1]. Since, as sketched before, the theoretical predictions are based on a rather solid foundation, they should not be discarded easily. Of course there is no proof that the broad $D/D^* + \pi$ systems are indeed the $j_q = 1/2P$ wave states; they could be radial excitations or non-resonant combinations of undetermined quantum numbers. Thus the DELPHI data taken by themselves are not necessarily in conflict with theoretical expectations.

However the plot thickens in several experimental as well as theoretical respects.

- In 2005 BELLE has presented an analysis of $\bar{B} \to l\bar{\nu}D/D^*\pi$ [38], which appears to be in conflict with previous findings. Reconstructing one B completely in $\Upsilon(4S) \to B\bar{B}$, one analyzes the decays of the other beauty meson and obtains

$$\mathrm{BR}\left(B^- \to l^-\bar{\nu}D\pi\right) = (0.81 \pm 0.18)\%\,, \qquad (46)$$
$$\mathrm{BR}\left(B^- \to l^-\bar{\nu}D^*\pi\right) = (1.00 \pm 0.22)\%\,, \qquad (47)$$
$$\mathrm{BR}\left(\bar{B}_d \to l^-\bar{\nu}D\pi\right) = (0.49 \pm 0.13)\%\,, \qquad (48)$$
$$\mathrm{BR}\left(\bar{B}_d \to l^-\bar{\nu}D^*\pi\right) = (0.97 \pm 0.22)\%\,. \qquad (49)$$

BELLE's separation of final states with D and D^* is of significant value, since it provides an indirect and model dependent handle on '3/2' and '1/2' production. For with the help of a quark model one can calculate both the production rates for the $D_{1,2}^{(3/2)}$ and $D_{0,1}^{(1/2)}$ and their branching fractions into $D\pi$ and $D^*\pi$. BELLE's numbers are actually quite consistent with the theoretical predictions the BT model yields for '3/2' P wave production. It is of course still desirable for BELLE to determine the quantum numbers of their hadronic final states.

Combining the two classes of final states BELLE arrives at

$$\mathrm{BR}\left(B^- \to l^-\bar{\nu}D^{(*)}\pi\right) = (1.81 \pm 0.20 \pm 0.20)\%\,, \qquad (50)$$

$$\mathrm{BR}\left(\bar{B}_d \to l^- \bar{\nu} D^{(*)}\pi\right) = (1.47 \pm 0.20 \pm 0.17)\,\%\,, \tag{51}$$

leaving room for a large $D^{(*)}\pi\pi$ component of $\sim (1.3 \pm 0.4)\%$, whereas previous studies have obtained 90% C.L. upper limits ranging from 0.35 to 1.3%; see (43) and (44).

– BABAR's most recent analysis [6] yields rather consistent numbers:

$$\mathrm{BR}\left(B^- \to l^- \bar{\nu} D\pi\right) = (0.63 \pm 0.09_{\mathrm{stat}} \pm 0.05_{\mathrm{syst}})\,\%\,, \tag{52}$$

$$\mathrm{BR}\left(B^- \to l^- \bar{\nu} D^*\pi\right) = (0.89 \pm 0.08_{\mathrm{stat}} \pm 0.06_{\mathrm{syst}})\,\%\,, \tag{53}$$

$$\mathrm{BR}(\bar{B}_d \to l^- \bar{\nu} D\pi) = (0.65 \pm 0.12_{\mathrm{stat}} \pm 0.05_{\mathrm{syst}})\%\,, \tag{54}$$

$$\mathrm{BR}\left(\bar{B}_d \to l^- \bar{\nu} D^*\pi\right) = (0.72 \pm 0.12_{\mathrm{stat}} \pm 0.06_{\mathrm{syst}})\,\%\,, \tag{55}$$

which can be combined to

$$\mathrm{BR}\left(B^- \to l^- \bar{\nu} D^{(*)}\pi\right) = (1.52 \pm 0.12 \pm 0.10)\,\%\,, \tag{56}$$

$$\mathrm{BR}\left(\bar{B}_d \to l^- \bar{\nu} D^{(*)}\pi\right) = (1.37 \pm 0.17 \pm 0.10)\,\%\,, \tag{57}$$

again leaving room for a significant $D^{(*)}\pi\pi$ component of about 1.3%.

– Using DELPHI's numbers stated in (15) and (45) and assuming that the "1" and "0" state decay 100% into $D^*\pi$ and $D\pi$, respectively, one arrives at

$$\mathrm{BR}\left(B^- \to l^- \bar{\nu} D\pi\right) = \mathrm{BR}\left(\bar{B}_d \to l^- \bar{\nu} D\pi\right) \sim (0.9 \pm 0.7)\% \tag{58}$$

$$\mathrm{BR}\left(B^- \to l^- \bar{\nu} D^*\pi\right) = \mathrm{BR}\left(\bar{B}_d \to l^- \bar{\nu} D^*\pi\right) \sim (1.9 \pm 0.4)\% \tag{59}$$

for a total of

$$\mathrm{BR}\left(B^- \to l^- \bar{\nu} D^{(*)}\pi\right) = \mathrm{BR}\left(\bar{B}_d \to l^- \bar{\nu} D^{(*)}\pi\right) \sim 2.8\%\,. \tag{60}$$

One should note that the qualitative trend is the same as with BELLE's findings, (46)–(49) – namely that $D^*\pi$ final states dominate over $D\pi$ ones – yet the total $D^{(*)}\pi$ rate exceeds that reported by BELLE.

– The BT model predicts for $D\pi$ and $D^*\pi$ production

$$\mathrm{BR}\left(B^- \to l^- \bar{\nu} D\pi\right) = \mathrm{BR}\left(\bar{B}_d \to l^- \bar{\nu} D\pi\right) = 0.51\%\,, \tag{61}$$

$$\mathrm{BR}\left(B^- \to l^- \bar{\nu} D^*\pi\right) = \mathrm{BR}\left(\bar{B}_d \to l^- \bar{\nu} D^*\pi\right) = 0.65\%\,, \tag{62}$$

which is on the low side of BELLE's numbers and a forteriori for DELPHI's findings, but consistent with BABAR's data. This is of course a rephrasing of the '1/2 versus 3/2' puzzle.

– In the BPS approximation [11, 12] one has $\tau_{1/2}^{(n)} = 0$. Assuming that the sum rule of (18) saturates already with the $n = 0$ state, one obtains $\tau_{3/2}^{(0)} = \frac{1}{2}$, leading to

$$\mathrm{BR}(B^- \to l^- \bar{\nu} D\pi) = \mathrm{BR}(\bar{B}_d \to l^- \bar{\nu} D\pi) = 0.39\%\,, \tag{63}$$

$$\mathrm{BR}(B^- \to l^- \bar{\nu} D^*\pi) = \mathrm{BR}(\bar{B}_d \to l^- \bar{\nu} D^*\pi) = 0.50\%\,; \tag{64}$$

i.e., lower still. This might not be that surprising, since the BPS ansatz is at best an approximation rather than a systematic expansion.

– So far there is no experimental evidence for high mass hadronic states. One finds that about 6.4% and 18.3% of all D^{**} states have masses between 2.6 and 3.3 GeV for the CDF and DELPHI data, respectively, which drop to 3.2% and 7.8% for the mass range 2.8 to 3.3 GeV and 0.3% and 3.1% for 3.0 to 3.3 GeV.
On the other hand, CDF seems to see more events below the $D^{3/2}$ peaks. Such low mass $D^{(*)}\pi$ events could be due to higher mass states decaying into $D^{(*)}\pi\pi$. CDF has not incorporated this scenario into their analysis, since previous measurements showed no evidence for such decays.

– One would conjecture that if the observed mass spectrum indeed differs significantly from theoretical expectations – in its center of gravity as well as its spread – then the measured hadronic mass moments should not follow theoretical predictions – yet they do [2–4, 35, 39, 40, 42].

– There is a more general problem. We have said before that the OPE treatment could a priori accommodate significant contributions with hadronic systems exhibiting a rather broad distribution in mass and centered below 2.5 or even 2.4 GeV. Yet closer scrutiny casts serious doubts on such a scenario. The OPE treatment involves applying quark–hadron duality. The latter can be expressed as saying that the rate evaluated on the quark–gluon level can be equated with the sum of observable exclusive hadronic channels, at least after some averaging or 'smearing' over energy scales has been applied; the latter is sometimes referred to as semi-local duality.
The question is: which exclusive channels could be seen as dual to contributions with hadronic mass below 2.4 or even 2.5 GeV? (i) The theoretical estimates agree that the $D^{1/2}$ states that can populate this mass range possess too small production amplitudes to contribute significantly; their amplitudes actually would have to be enhanced greatly to overcome their kinematic suppression in semileptonic B decays. (ii) There are many higher orbital excitations of course, and taken together they might have a 'fighting' chance to yield a significant contribution to $\Gamma_{\mathrm{SL}}(B)$ – yet they all lie above 2.5 GeV in mass. It would not correspond to our usual picture that exclusive channels all above 2.5 GeV are dual to quark-level contributions computed to all lie below it. (iii) There is one intriguing possibility. We know of one basic failure of all quark models: they cannot explain the mass of the baryonic Roper resonance. For all quark

models predict the mass of the first radial excitation to be higher than that for the first orbital excitation – an inequality clearly reversed for the Roper resonance. Would it be possible that the spectrum of charm resonances exhibits an analogous effect for mesons meaning that radial excitations can lie below the P wave states discussed before, and they make up the D_X component? This would be a most intriguing – yet also most exotic explanation.

In summary: ALEPH, DELPHI and D0 agree in finding a rate of about 0.8–1% of Γ_B for the production of the two narrow D^{**} states combined. This value is quite consistent with theoretical expectations for the $D^{3/2}$ rates. BELLE's data also fit naturally into this picture. The problem arises in the production of the broad D^{**} states: the rates found by ALEPH and DELPHI suffice to saturate $\Gamma_{\rm SL}(B)$, yet they exceed the predictions for $\bar{B} \to l\bar{\nu}D^{1/2}$ by about an order of magnitude. BELLE's numbers on the other hand agree reasonably well with predictions, yet they fall short of saturating $\Gamma_{\rm SL}(B)$. The mass distributions of the broad D^{**} states, for which there is no clear experimental verdict, might pose a theoretical conundrum: if it is centered below 2.5 GeV, we have no natural candidates for these states.

8 Comments on nonleptonic B decays

It is usually argued – with very valid reasons – that the theoretical description is much murkier for nonleptonic than semileptonic B decays. We might encounter here one of the few exceptions to this general rule of thumb, and we have been alluding to this possibility already. One can analyze the inclusive transition

$$\bar{B} \to \pi X_c \tag{65}$$

and study the hadronic system X_c in the spirit of factorization; i.e., one analyzes its recoil mass spectrum, its quantum numbers and decay characteristics with the following motivation, (i) The higher complexity of nonleptonic dynamics can be seen as an actually advantage here, for it provides additional production scenarios. More specifically it allows for significant production of $D^{1/2}$ states through the W emission diagram, which is not possible in semileptonic transitions. (ii) Without a neutrino in the final state it might be easier to determine the mass distributions and quantum numbers of X_c.

As already mentioned, for $X_c = D^*$, $D^{3/2,1/2}$ the theoretical predictions from the BT model have been found in reasonable, though not compelling agreement with the data.

One wants to extend such studies to the fully inclusive case and analyze also the higher mass D^{**} configuration. Intriguing first steps in this direction have already been taken by BABAR [25, 41]. The recoil mass spectra obtained by BABAR show clear D and D^* peaks for charged and neutral B decays. The former exhibit also a clear peak around 2.4–2.5 GeV and maybe a signal also in the region above 2.6 GeV. In the $\bar{B}_d$ case there might be a signal in the 2.4–2.6 GeV region, but not much else. The peaks in the 2.4–2.5 GeV region are natural candidates for showing $D^{3/2}$ production. The verdict on the domain beyond 2.5 GeV, which could be populated by the same configurations as in $\bar{B} \to l^-\bar{\nu}D^{**}_{\rm broad}$, is tantalizing inconclusive.

9 Conclusions and a call for action

The B_d and B_u inclusive semileptonic widths have been well measured. Most if not even all of it has been identified in $\bar{B} \to l\bar{\nu}D/D^* + (0,1)\pi$. The theoretical description of $\bar{B} \to l\bar{\nu}X_c$ rests on solid foundations. The potential discrepancies discussed in this note, which affect at most 20% of the semileptonic B transitions, cannot lead to a significant increase in the uncertainty with which $|V(cb)|$ can be extracted from $\Gamma(\bar{B} \to l\bar{\nu}X_c)$. On the other hand they should not be 'brushed under the rug'. Theory does make non-trivial predictions of a rather sturdy nature. The OPE treatment is genuinely based on QCD, and while the BT description invokes a model, it implements QCD dynamics for heavy flavor hadrons to a remarkable degree. These predictions therefore deserve to be taken seriously and not discarded at the first sign of phenomenological trouble. Preliminary lattice studies show no significant enhancement of '1/2' production. The numbers we have given for the theoretical expectations should be taken with quite a few grains of salt. Yet the predicted pattern that the abundance of '3/2' P wave resonances dominates over that for '1/2' states in semileptonic B decays is a robust one. Even a failure of such well-grounded predictions could teach us valuable lessons on non-perturbative dynamics and our control over them; it would certainly provide a valuable challenge to lattice QCD. Yet there is more: we know that D, D^* and $D^{3/2}$ production do not saturate $\Gamma_{\rm SL}(B)$, and theory tells us that $D^{1/2}$ cannot contribute significantly. What is then the nature of the missing hadronic configurations?

On the experimental side the next important steps are as follows.

- In some of our discussion above we have modeled $\Gamma_{\rm SL}(\bar{B})$ as the incoherent sum of $\bar{B} \to l^-\bar{\nu}D/D^*/D^{3/2}/D_X$ to infer the average mass of the configuration D_X and its variance from the measured hadronic mass moments. The values of $\langle M(D_X)\rangle$ and $\sqrt{\langle M^2(D_X)\rangle - \langle M(D_X)\rangle^2}$ serve as very useful diagnostics of the underlying dynamical situation. One finds that $\langle M(D_X)\rangle$ varies from 2.4 to 2.6 – or even 2.7 – GeV, depending on whether one uses the branching ratios of (6) or of (5). The main reason for this relatively sizable shift is the variation in $\mathrm{BR}(\bar{B} \to l^-\bar{\nu}D^*)$. It would be most helpful to have this branching ratio determined more precisely.
- Clarifying the size, mass distribution and quantum numbers of $\bar{B} \to l\bar{\nu}[D/D^*\pi]_{\rm broad}$ and searching for $\bar{B} \to l\bar{\nu}D/D^* + 2\pi$ with even higher sensitivity.
- The data should be presented separately for $\bar{B} \to l\bar{\nu}D + \pi$s and $\bar{B} \to l\bar{\nu}D^* + \pi$s, since it provides more theoretical diagnostics.

– More detailed analysis of $\bar{B} \to \pi X_c$, in particular in the high mass region for the X_c and separately for charged and neutral B decays [25, 41].

These are challenging experimental tasks, yet highly rewarding ones as well.

– They probe our theoretical control over QCD's nonperturbative dynamics in novel and sensitive ways. This is an area where different theoretical technologies – the OPE, quark models and lattice QCD – are making closer and closer contact.
  The lessons to be learnt will be very significant ones, no matter what the eventual experimental verdict will be.
  – A confirmation of the OPE expectations and even the more specific BT predictions would reveal an even higher degree of theoretical control over nonperturbative QCD dynamics than has been shown through $\Gamma(\bar{B} \to l\bar{\nu}X_c)$.
  – Otherwise we could infer that formally nonleading $1/m_Q$ corrections are highly significant numerically. Those corrections had to be highly enhanced to overcome the kinematic suppression in the production of $D^{1/2}$ in semileptonic B decays.
    Such an insight would be surprising – yet important as well. In particular it would provide a highly nontrivial challenge to lattice QCD. Meeting this challenge successfully would provide lattice QCD with significantly enhanced validation.

The call for further action is directed to theorists as well.

– In the BT model one can compute the production rates for the higher orbital and radial excitations in semileptonic B decays. The individual rates seem to be rather small. It is conceivable that summing over a multitude of them yields a significant contribution.
– The BT model predictions were obtained in the heavy quark limit. Corrections to this limit could be quite important as suggested in [27], and they could significantly change the relative weight of $\tau_{1/2}^{(n)}$ and $\tau_{3/2}^{(n)}$. Calculating or at least constraining those corrections would be a most worthwhile undertaking – alas it requires some new ideas. A priori one can conceive of different ways of extending the BT description to include finite mass effects, yet they are unlikely to be equivalent. The foundations for a promising way have been laid in [17].
  The authors of [43, 44] find significant rates for the production of radial excitation that are enhanced further by $1/m_Q$ corrections. While we cannot quite follow their argumentation, $1/m_Q$ corrections due to potential exchange diagrams could indeed be sizable. This intriguing possibility needs and deserves intense scrutiny. There is also the exotic possibility that one finds here a mesonic analogue to the Roper resonance, namely that the radial (and even other) excitations of charm mesons are significantly lower in mass than predicted by quark models. This would presumably raise their production rates in semileptonic B decays to the sought-after level.
– Lattice QCD studies of '1/2' and '3/2' production in semileptonic B decays has to be pursued with vigour. Such studies could turn out to be veritable 'gold mines' as far as validation is concerned. One can evaluate the spectrum of the higher radial and orbital excitations D^{**}, for which some encouraging results have already been obtained [28, 29]. Lattice calculations at finite values of m_c should be performed, which would teach us about $1/m_c$ corrections.
– The strong decays $D^{**} \to D/D^* + \pi\pi$ should be estimated using heavy quark symmetry arguments augmented by quark model considerations.

A final comment: the experimental analyses we advocate require considerable efforts. We strongly believe that such an effort is mandated by the insights to be gained, even if they are of a subtle nature.

– They can provide us with important insights into the workings of nonperturbative dynamics. Lessons on the significance of $1/m_Q$ corrections and on systematic short comings of quark models would be of general value for the theoretical control we can establish over heavy flavor dynamics.
– They probe the subtle concept of quark–hadron duality in novel ways.
– The greatest practical gain might emerge for lattice QCD: if the latter can meet the challenge of such detailed data successfully, it would have gained a qualitatively new measure of validation.

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